RHEOELECTRIC CONVERTER CALCULATION

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The problem of steady-state rotation of a cylindrical dielectric rotor with metal core within a weakly conductive liquid in an electric field is solved.

Steady-state rotation of a cylindrical homogeneous rotor of dielectric material located in a liquid in an electric field was considered in [1-5], since this effect is used in dielectric motors [5]. Use of a metal core in the dielectric rotor reduces the critical electric field value and increases the turning moment of the motor [1].

Figure 1 shows a diagram of an infinite cylinder rotating at angular velocity ω about the z axis in an infinite viscous weakly conductive liquid. The constant external electric field is homogeneous at infinity and parallel to the x axis. The outer radius of the rotor will be denoted by a_2, while a_1 is the radius of the inner portion of the rotor formed of ideally conductive material. The material forming the annular part of the rotor has low electrical conductivity k_2 and dielectric permittivity ε_2. In order to determine the conditions necessary for steady-state rotation, it is necessary to determine the electric field in the rotor and liquid, calculate the turning moment, and equate this moment to the viscous resistive forces applied to the rotor by the liquid [2-4].

In the cylindrical coordinate system r, θ, z, for the electric field we have [2, 3]:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\varphi}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2\varphi}{\partial\theta^2}=0,\ a_2\leqslant r\leqslant\infty, \tag{1}$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\varphi_2}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2\varphi_2}{\partial\theta^2}=0,\ a_1\leqslant r\leqslant a_2, \tag{2}$$

$$\varphi_1=0,\ 0\leqslant r\leqslant a_1. \tag{3}$$

With the assumptions made above and in the absence of both intrinsic equilibrium charge and a constant dipole moment within the dielectric, the boundary conditions for Eqs. (1), (2) have the form [2, 3]:

$$\varphi|_{r\to\infty}=-E_0 r\cos\theta, \tag{4}$$

$$\varphi(a_2,\ \theta)=\varphi_2(a_2,\ \theta),\ \varphi_2(a_1,\ \theta)=0, \tag{5}$$

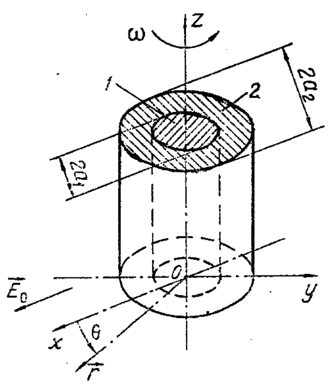

Fig. 1. Diagram of composite rotor: 1) cylindrical rotor section of ideally conductive metal; 2) dielectric shell.

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$$\varepsilon_2 \frac{\partial \varphi_2}{\partial r}\bigg|_{r=a_2} - \varepsilon \frac{\partial \varphi}{\partial r}\bigg|_{r=a_2} = \frac{1}{\varepsilon_0}\sigma, \tag{6}$$

$$I(\theta) = \sigma(\theta)\,\omega a_2, \quad i = -\frac{1}{a_2}\frac{\partial I}{\partial \theta}, \tag{7}$$

$$i = k_2 \frac{\partial \varphi_2}{\partial r} - k \frac{\partial \varphi}{\partial r}. \tag{8}$$

To solve Eqs. (1), (2) with boundary conditions (4)-(8) it is necessary to determine six arbitrary constants A, B, C_2, D_2, A_2, B_2 in the expressions [2]:

$$\varphi = -E_0 r\cos\theta + \frac{A\cos\theta}{r} - \frac{B\sin\theta}{r},$$
$$\varphi_2 = C_2 r\cos\theta + D_2 r\sin\theta + \frac{A_2\cos\theta}{r} - \frac{B_2\sin\theta}{r}. \tag{9}$$

Substitution of Eq. (9) in Eqs. (4)-(8) gives

$$B = \frac{2\varepsilon_0 E_0 \omega a_2^2 (1 - h^4)(k\varepsilon_2 - \varepsilon k_2)}{(1 + \tau^2\omega^2)[k_2(1 + h^2) + k(1 - h^2)]^2}, \tag{10}$$

where

$$\tau = \frac{\varepsilon_0[\varepsilon_2(1 + h^2) + \varepsilon(1 - h^2)]}{k_2(1 + h^2) + k(1 - h^2)}, \quad h = \frac{a_1}{a_2}. \tag{11}$$

The remaining constants in Eq. (9) are related to B by the following expressions:

$$A = \frac{\varepsilon_0\varepsilon\omega(a_1^2 - a_2^2) - \varepsilon_0\varepsilon_2\omega(a_1^2 + a_2^2)}{k(a_1^2 - a_2^2) - k_2(a_1^2 + a_2^2)} B + \frac{E_0 a_2^2[k(a_2^2 - a_1^2) - k_2(a_1^2 + a_2^2)]}{k(a_1^2 - a_2^2) - k_2(a_1^2 + a_2^2)}, \tag{12}$$

$$A_2 = \frac{Aa_1^2 - E_0 a_1^2 a_2^2}{a_1^2 - a_2^2}, \quad B_2 = \frac{a_1^2}{a_1^2 - a_2^2}, \tag{13}$$

$$C_2 = \frac{E_0 a_2^2 - A}{a_1^2 - a_2^2}, \quad D_2 = \frac{B}{a_1^2 - a_2^2}. \tag{14}$$

The moments of the electric forces and the liquid viscous friction forces per unit rotor length are equal to [2, 3]:

$$L^{e} = \frac{4\pi\varepsilon\varepsilon_0 E_0}{2} B, \quad L^{v} = -4\pi\eta a_2^2\,\omega. \tag{15}$$

Substituting Eq. (15) in the dynamic condition for existence of steady-state rotor rotation

$$L^{e} + L^{v} = 0, \tag{16}$$

we find ω and E_C in the form

$$\omega^2 = \frac{\beta E_0^2 - N(k_2 + k\beta)^2}{N\varepsilon_0^2(\varepsilon_2 + \varepsilon\beta)^2}, \tag{17}$$

$$E_C^2 = N\frac{(k_2 + k\beta)^2}{\beta}, \tag{18}$$

where

$$N = \frac{\eta}{\varepsilon_0^2\varepsilon(k\varepsilon_2 - \varepsilon k_2)}, \quad \beta = \frac{1 - h^2}{1 + h^2}. \tag{19}$$

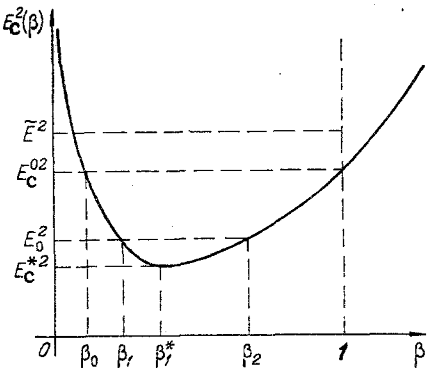

Fig. 2

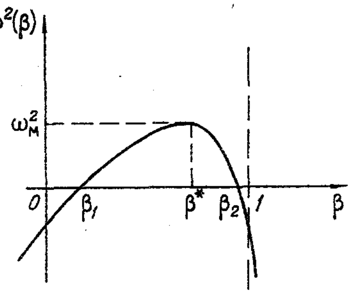

Fig. 3

Fig. 2. Square of critical external electric field E_C^2 vs quantity $\beta = (1-h^2)/(1+h^2)$, which characterizes ratio of radius a_1 (ideally conductive portion of rotor) to radius a_2 of composite rotor; $h = a_1/a_2$.

Fig. 3. Square of angular velocity ω^2 of composite rotor vs quantity β for $E_C^* < E_0 \leqslant E_C^0$.

Here E_C is the critical value of external electric field intensity, which when exceeded causes the composite rotor to rotate.

The necessary conditions for existence of steady-state rotation of the composite rotor are [2]

$$E_0 > E_C(\beta),\quad k/\varepsilon > k_2/\varepsilon_2. \tag{20}$$

We note that at $\beta = 1$ Eqs. (17), (18) transform to the corresponding expressions of a homogeneous dielectric rotor [1, 3]:

$$\omega_0^2 = \frac{\varepsilon_0^2 \varepsilon E_0^2 (k\varepsilon_2 - k_2\varepsilon) - \eta (k_2 + k)^2}{\varepsilon_0^2 \eta (\varepsilon + \varepsilon_2)^2}, \tag{21}$$

$$E_C^{0^2} = \frac{\eta (k_2 + k)^2}{\varepsilon_0^2 \varepsilon (k\varepsilon_2 - \varepsilon k_2)}. \tag{22}$$

Study of the behavior of the functions $\omega = \omega(\beta)$, $E_C = E_C(\beta)$ in Eqs. (17), (18) with the limitations of Eq. (20) reveals that use of a composite rotor leads to increase in ω and decrease in E_C only with an appropriate choice of shell (Figs. 2 and 3). In the opposite case the velocity and power characteristics of the converter may be degraded. The following cases are possible.

1. If $\varepsilon_2 > \varepsilon$, $k > k_2$, then E_C (Fig. 2) may be either smaller or larger than E_C^0, since the following inequalities are fulfilled:

$$E_C \leqslant E_C^0 \ \text{at} \ \beta_0 \leqslant \beta < 1,\ \beta_0 = k_2^2/k^2; \qquad E_C > E_C^0 \ \text{at} \ \beta < \beta_0. \tag{23}$$

In this case on the interval $0 \leqslant \beta \leqslant 1$ the function $E_C(\beta)$ has a local minimum at $\beta = \beta^*_1$:

$$E_C^*(\beta_1^*) = (4Nkk_2)^{1/2},\quad \beta_1^* = k_2/k. \tag{24}$$

Depending on the value of β chosen, the composite rotor angular velocity ω may be either higher or lower than the angular velocity of a corresponding homogeneous rotor ω_0 (Fig. 3).

From Eqs. (17)-(20) we have the inequality

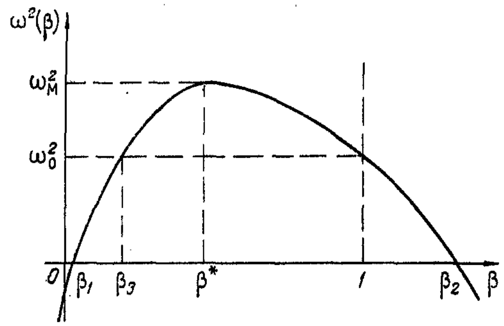

Fig. 4. Square of composite rotor angular velocity ω^2 vs β for $E_c^0 \leqslant E_0 < \tilde{E}$.

$$\omega > 0 \text{ at } E_c^* < E_0 \leqslant E_c^0,\ \beta_1 < \beta < \beta_2,$$
$$\beta_{1,2} = \frac{2Nkk_2 - E_0^2 \pm \sqrt{E_0^2 - 4Nkk_2}}{-2Nk^2},\ \beta_2 > \beta_1 > 0. \tag{25}$$

It follows from Eq. (25) that at $E_0 \leqslant E_c^0$ the homogeneous rotor does not move, while the composite rotor will turn, while in the segment $\beta_1 \leqslant \beta \leqslant \beta_2$ ω will reach a local maximum ω_M:

$$\omega_M^2 = \frac{\beta^* E_0^2 - N(k_2 + k\beta^*)^2}{N\varepsilon_0^2(\varepsilon_2 + \varepsilon\beta^*)^2},$$
$$\beta^* = \frac{E_0^2\varepsilon_0\varepsilon\varepsilon_2 - 2\eta k_2}{E_0^2\varepsilon_0^2\varepsilon^2 + 2\eta k},\quad h^* = \left(\frac{1-\beta^*}{1+\beta^*}\right)^{\frac{1}{2}} \tag{26}$$

The angular velocity of the composite rotor will be higher than that of the homogeneous rotor, if the condition

$$\omega \geqslant \omega_0 \text{ at } E_c^0 \leqslant E_0 < \tilde{E},\ \beta_3 \leqslant \beta < 1,$$
$$\tilde{E} = [2N(k + k_2)(k\varepsilon_2 - \varepsilon k_2)/(\varepsilon_2 - \varepsilon)]^{1/2} \tag{27}$$

is satisfied (Fig. 4), where β_3 is the positive root of the equation

$$(F\varepsilon^2 + NGk^2)\beta_3^2 + (2F\varepsilon_2\varepsilon + 2GNk_2k - GE_0^2)\beta_3 + GNk_2^2 + F\varepsilon_2^2 = 0,$$
$$F = E_0^2 - N(k_2 + k)^2,\ G = (\varepsilon_2 + \varepsilon)^2. \tag{28}$$

According to Eq. (27), at $\beta_1 < \beta < \beta_3$ the composite rotor will rotate more slowly than the corresponding homogeneous one, and will remain motionless at $0 < \beta \leqslant \beta_1$. A similar regime is realized if the applied field intensity E_0 exceeds the upper critical value $\tilde{E}$.

2. If $\varepsilon > \varepsilon_2 > \varepsilon k_2/k$, $k > k_2$, then to calculate the composite rotor parameters all the expressions and conclusions of the previous section are valid, with the angle difference that in this case, $\tilde{E} = \infty$.

3. If $\varepsilon_2 > \varepsilon$, $k_2 > k > k_2\varepsilon/\varepsilon_2$, then E_c of the composite rotor is always greater than E_c^0, the critical value of applied electric field for the corresponding homogeneous dielectric rotor. In this case β_1^* in Fig. 2 lies in the interval [1, ∞]. In this case there is no shell thickness which will produce an increase in speed over a homogeneous rotor. The curve $\omega^2(\beta)$ (Fig. 4) is shifted to the right, so that β^* is located in the interval [1, ∞].

We will offer examples of ω and E_c calculations for composite rotors. The rotor shell will be Plexiglas ($k_2 = 10^{-13}\ \Omega^{-1}\cdot m^{-1}$, $\varepsilon_2 = 2.4$) with transformer oil as the liquid ($\varepsilon =$

2.3, $\eta = 0.032$ N · sec · m^{-2}, $k = 10^{-9}\,\Omega^{-1}\cdot$ m^{-1}). From Eq. (18) at $\beta = 1$ we find that $E_c^o = 271.97 \cdot 10^3$ V/m. Let $E_0 = E_c^o$, so a homogeneous Plexiglas rotor will not move. From Eq. (26), for the composite rotor we find $h^* = 0.703$, with maximum angular velocity possible at $E_0 = 271.97 \cdot 10^3$ V/m of $\omega_M = 16.81$ sec^{-1}, which corresponds to $n_M = 160.5$ rpm.

Let us take a porcelain shell ($k_2 = 3 \cdot 10^{-13}\,\Omega^{-1}\cdot$ m^{-1}, $\varepsilon_2 = 6$) with liquid consisting of a 27% solution of isopropyl alcohol in toluol ($k = 1.1 \cdot 10^{-7}\,\Omega^{-1}\cdot$ m^{-1}, $\varepsilon = 4.82$, $\eta = 10^{-3}$ N · sec · m^{-2}). Now $E_c^o = 220.27 \cdot 10^3$ V/m, $E^* = 772.05$ V/m, i.e., the minimum critical electric field value for the composite rotor is 285 times smaller than the critical value for a homogeneous porcelain rotor. If, for example, $E_0 = 16.05 \cdot 10^4$ V/m, then at $h = 0.6$, we find from Eq. (17) that the homogeneous porcelain rotor will not rotate, while the composite one will turn at angular velocity $\omega = 22$ sec^{-1}, corresponding to $n \simeq 210$ rpm.

NOTATION

ω, angular velocity; k_1, k_2, resistivities of liquid and rotor dielectric shell; ε, ε_2, dielectric permittivities of liquid and shell material; φ, φ_2, φ_1, electric field potentials in liquid, dielectric shell, and ideally conductive rotor core; E_0, applied electric field intensity far from rotor; i, discontinuity in normal component of electric field density vector on rotor surface; σ, electron charge surface density; $I(\theta)$, convection current on rotor surface; η, liquid viscosity; E_c, E_c^o, critical value of applied electric field intensity for composite and homogeneous rotors; ω_0, angular velocity of homogeneous dielectric rotor rotation; E^*_M, minimum E_c value; ω_M, maximum value of ω; $\varepsilon_0 = 8.85416 \cdot 10^{-12}$ F/m.

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